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Resurgence in Quantum Field Theory

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ABSTRACT: Outline of topics covered in lecture series on resurgence in quantum field theory at Tokyo Metropolitan University Summer 2022. Full handwritten notes are also available.

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1 Lecture 1

1.1 Overview

- Interdisciplinary topic. I will assume zero knowledge of either area.
- Why is resurgence studied in the context of quantum field theory?
- What can quantum field theory bring to resurgence? A lot of examples.
- Deeper connections — BPS state counting, spectral networks etc.

1.2 The plan

- Broad discussion (a fairy tale story) of connections between resurgence and quantum fields
- Introduction to quantum field theory with concrete examples
 - Quantum mechanics as a qft
- Review of resurgence and perturbation theory
 - Ecallé's correspondence and Borel resummation
- Applications (unlikely to have time to cover all)
 - BPS states in 4d supersymmetry
 - Large N matrix models
 - Dunne and Unsal's Cheshire cat resurgence
 - Exactly soluble Yang-Mills at large N

1.3 Introduction

- Transseries
- Classical field theory
- Yang-Mills gauge theory and instantons
- Feynman diagrams
- Supersymmetry
- BPS state counting and exact WKB

1.4 What is quantum field theory?

- Ingredients of QFT
- 0d Feynman diagram example

2 Lecture 2

2.1 Resurgence for physicists

- Correspondence between complex analysis and asymptotics
- Transseries and resurgent functions
- Resurgence lemma

2.2 Quantum field theory upgrade

- One dimensional QFT = quantum mechanics
- Boundaries and Hilbert spaces
- Tunnelling

2.3 Lighting quantum mechanics review

- States and measurements
- Observables
- Symmetries
- Transition amplitudes
- Some examples
- The partition function with harmonic oscillator example

3 Lecture 3

3.1 Fermions and supersymmetry

- Structure of supersymmetric Hilbert space
- Witten index
- Supersymmetric path integral
- 1d quantum mechanics example
- Localisation principle

3.2 Resurgence in supersymmetric quantum mechanics

References